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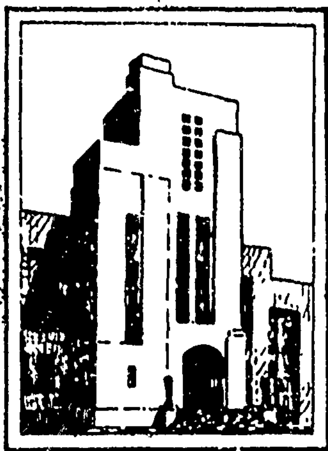
# DAVID TAYLOR MODEL BASIN



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## A STUDY OF THE HYDRODYNAMIC LOADS ON A CYLINDRICAL MAST AND RADOME

by

Raphael D. Cahn



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HYDROMECHANICS LABORATORY

**RESEARCH AND DEVELOPMENT REPORT**

May 1958 Report 1239

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## SYMBOLS

| | | |
|---|---|---|
| $C_d$ | = | coefficient of drag |
| $C_l$ | = | coefficient of lift |
| $D$ | = | drag force |
| $E$ | = | modulus of elasticity |
| $I$ | = | moment of inertia |
| $M_b$ | = | mass of beam |
| $M_l$ | = | mass of load |
| $R$ | = | Reynolds number |
| $S$ | = | Strouhal number |
| $V$ | = | velocity |
| $d$ | = | diameter |
| $f$ | = | frequency |
| $l$ | = | length |
| $\nu$ | = | kinematic viscosity |
| $\rho$ | = | mass density |
| $\omega$ | = | angular frequency |

## ABSTRACT

The predominant forces on a mast projecting from a submerged submarine through the surface are steady drag, periodic lift, and wave impact. As the natural frequency of vibration of the mast-radome configuration under consideration is in the vicinity of the vortex shedding frequency, a catastrophic instability should be expected. Since the function of the radome at the top of the mast is to protect the encapsulated instruments, any large deflection of the dome could endanger the contents. The forces on the dome due to drag and breaking waves are estimated for particular operating conditions, numerical values being presented in tabular form for the force distribution over the dome. Also given is the frequency of occurrence of the significant wave for these conditions. The impact force prevails over all others and represents the major obstacle to successful operation of the system in high seas. Pertinent criteria are established to serve as guidelines for the design of the system.

## INTRODUCTION

One of the navigation devices for a submarine is the Radiometric Sextant, a radio telescope housed in a radome. In making a submerged, underway observation, the radome is extended by a mast from a topside storage chamber and is projected through the surface in periscope fashion. In this position the mast-radome combination may be subjected to a variety of hydrodynamic loads:

steady drag and lift, oscillatory drag and lift, and wave action. The response of the radome and mast to severe conditions of loading, such as high seas or crash dives, determines the operational capabilities. Also it is important to know the period of time that the radome can be expected to remain exposed to the air continuously. The David Taylor Model Basin has been requested by the Bureau of Ordnance to analyze the configuration shown in Drawing No. 11824 of the Detroit Controls Corporation to determine the factors affecting the performance of the system. (See Figure 1.)

## ANALYSIS

The two shapes for the radome to be studied are pictured in Figure 1. For the purpose of an approximate calculation, the cylindrical mast is assumed to be infinitely long compared with its diameter, and deeply submerged. The Reynolds numbers for the mast shown in Figure 1 are computed from $R = Vd/\nu$

At 6 knots $R_6 = \frac{6 \times 1.69 \times 1.5}{1.4 \times 10^5} = 1.1 \times 10^6$

At 20 knots $R_{20} = \frac{20 \times 1.69 \times 1.5}{1.4 \times 10^5} = 3.6 \times 10^6$

## DRAG FORCES

Drag coefficients for this cylinder are taken from Reference 1.*
The drag is assumed to be uniform along the length, and is equal to $D = C_d \frac{1}{2} \rho V^2 d$.

At 6 knots $D = 0.4 \times \frac{1}{2} \times \frac{64}{32.2} (6 \times 1.69)^2 \times 1.5$

$= 61 \text{ lb/ft}$

* References are listed on page 9.

At 20 knots $D = 0.5 \times \frac{1}{2} \times \frac{64}{32.2}(20 \times 1.69)^2 \times 1.5$

$= 850 \; lb/ft$

For the radome the Reynolds number is:

At 6 knots $R = \frac{6 \times 1.69}{1.4 \times 10^{-5}} \times \frac{57}{12} = 3.5 \times 10^6$

At 20 knots $R = \frac{20 \times 1.69}{1.4 \times 10^{-5}} \times \frac{57}{12} = 1.2 \times 10^7$

Drag coefficients at these very high Reynolds numbers have never been determined, even for such simple shapes as spheres or infinite cylinders. The same is true for the pressure distributions about these objects. Therefore, to obtain approximate values for drag and pressure coefficients certain simplifying assumptions have been made:

a. The drag and pressure coefficients at the midpoint of a finite cylinder are the same as for an infinite cylinder.

b. The drag and pressure coefficients for the flow about spheres and cylinders do not change with Reynolds number for Reynolds numbers above $1.0 \times 10^6$ .

c. The pressure coefficients at the intersection of a hemisphere and a cylinder are the average values for the sphere and cylinder.

d. The pressure coefficients vary linearly along the geometrical elements.

e. The oncoming flow is perpendicular to the axis.

f. Local deformations caused by the flow result only in second-order changes to the flow.

Using these assumptions, an array of concentrated forces at points over the surface of the hemisphere-cylinder combination can be determined.

The coordinates of the points were selected in the following manner: (Refer to Figure 1.) Vertical elements are spaced 15 degrees around the body, intersecting 15-degree "parallels" in the hemisphere and six equally spaced circular elements in the cylinders. For the spherical body the force distributions for a sphere is assumed, with meridians spaced at 15 degrees and cut by parallels spaced 15 degrees apart. The pressure distributions of cylinders and spheres, taken from Reference 2, is shown in Figure 2. This figure also shows the pressure distribution about the elliptical forward portion of two sonar domes.[3] Since the forward half of the sphere-cylinder radome may be considered to be an ellipse of zero eccentricity, these pressure distributions should be interrelated. The values of the maximum negative pressure, a significant function of the shape, are plotted against the minor to major axis ratio (Figure 2). The points lie close to a straight line, which suggests that a similarity does exist and that the distribution of pressure chosen for the radome is probably realistic. Also shown in Figure 2 is a third pressure distribution on the ellipses near the junction of the cylindrical and ellipsoidal portion. This line lies between the other two lines that represent the pressures around the cylindrical and ellipsoidal portions. This seems to justify assumption (c). Tables 1 through 3 present the force distributions over the radomes at 6 and 20 knots.

LIFT FORCES

When the radome is protruding from its storage container, only this upper portion is exposed to the flow and therefore subject to vertical lift forces. If the vertical components of the forces over the spherical portion

of the dome are summed, the following lifts are obtained:

At 6 knots 765 lb, upward

At 20 knots 8500 lb, upward.

Buoyancy forces give additional lifts as follows:

For Figure 1a 5200 lb, upward

For Figure 1b 3600 lb, upward

For the mast 113 lb per ft, upward.

## VIBRATION

If the mast is considered as a cantilever beam in air, fixed at the upper bearing and having an added mass at the free end, the natural frequency of the first mode is given by Reference 4 as

$$\omega^2 = \frac{3.03\ EI}{l^3(M_l + .23 M_b)}$$

$$M_l = \frac{1000}{32.2 \times 12} = 2.82 \text{ lb sec}^2 \text{ in}^{-1}$$

$$0.23 M_b = 490 \times \frac{\pi}{4} \times \frac{(18^2-16^2)}{144} \times \frac{17.5}{32.2 \times 12} \times 0.23 = 1.615 \text{ lb sec}^2 \text{ in}^{-1}$$

$$l = 17.5 \times 12 = 210 \text{ in} \qquad l^3 = 9.26 \times 10^6 \text{ in}^3$$

$$E = 3 \times 10^7 \text{ lb in}^{-2}$$

$$I = \frac{\pi}{64}(D_o^4 - D_i^4) = \frac{\pi}{64}(18^4 - 16^4) = 2.11 \times 10^4 \text{ in}^4$$

$$\omega^2 = \frac{3.03 \times 3 \times 10^7 \times 2.11 \times 10^4}{9.26 \times 10^6 \times 4.435} = 852 \text{ rad}^2 \text{ sec}^{-2}$$

$$f = \frac{\omega}{2\pi} = \frac{\sqrt{852}}{2\pi} = 4.7 \text{ cps}$$

A characteristic of the flow about a cylinder is the periodic shedding of vortices. The freqaency of the exciting force from this hydrodynamic source can be computed from the Strouhal number at the given Reynolds number, using the equation $f = VS/d$. A value for $S$ of 0.4 is given in Reference 1 for Reynolds numbers between 1 and $3 \times 10^6$.

At 6 knots $f = \frac{1.69 \times 6 \times 0.4}{1.5} = 2.7$ cps

At 20 knots $f = \frac{1.69 \times 20 \times 0.4}{1.5} = 9.0$ cps

Thus it can be seen that the (rigid) mass is subjected to an alternating transverse lift force in the neighborhood of its natural frequency. The mast, therefore, would be expected to vibrate at its natural frequency. Vortices would continue to be shed, not at the frequency given by the Strouhal number, however, but at the natural frequency of the mast. By this mechanism of self-excitation, very large periodic lift forces and concomitant large amplitudes of vibration are produced. The amplitude of the oscillating lift force is given by

$$L = C_l \times \tfrac{1}{2} \rho V^2 A$$

where the amplitude of $C_l$ is equal to 0.6 for nonvibrating cylinders.[5] For a vibrating cylinder the coefficient is somewhat higher, and depends upon the dissipation and radiation of the energy.[6] A value of 1.0 has been reported for a cylinder towed in water.[7] The periodic drag forces, which are a part of this same mechanism, result in a relatively small change in drag and so their effect may be neglected.

## WAVE ACTION

Wave impact loads and exposure time have been studied in connection with another submarine navigation device.[8] The general problem of impact loading is discussed in Reference 9. In Reference 8 a ship speed of 6 knots into a State 5 sea is assumed. The effective dynamic pressure is shown to be 1820 lb/sq ft, and with an impact coefficient of 3.14 gives a unit impact pressure of 5715 lb/sq ft. The significant wave height for a wind speed of 24 knots is 12 ft. For a breaking wave the wave length would be 120 ft. The time between successive encounters with such a wave is 38 seconds. This is based on an average wave length of 160 ft, an average wave height of 7.9 ft, and a range of periods from 3.7 to 13.5 seconds. If, however, the ship is assumed to go at 6 knots with the sea, the effective pressure is 506 lb/sq ft, giving an impact pressure of 1590 lb/sq ft. Here the time between encounters would be 96 seconds.

## DISCUSSION

Of the forces encountered by the mast, the drag appears to be the least forbidding. The forces on the mast due to vortex shedding and wave action, as well as the drag, could be reduced by fairing. However this is not practicable, since, in general, the directions of the forces cannot be predicted. For the dome, however, the forces are very large whereas the structure is relatively light. Local distortion of the surface will affect the internal clearances but the real problem is the deflection of the dome, as a whole, due to drag or wave action. Tables 1, 2, and 3 can be used to estimate the drag and lift forces acting on the domes. Although the forces

on the spherical dome are smaller, this advantage is offset by the narrower base and so a larger deflection should be expected. In a State 5 sea the dome would certainly be expected to encounter at least one breaking wave. Since streamlining is not feasible, the dome must have built-in rigidity, a requirement which may not be compatible with its function.

## CONCLUSIONS

In relatively calm seas the stresses in the mast due to drag and vibratory forces predominate. They can be minimized by making the mast as stiff as possible; i.e., keeping the unsupported length short and the moment of inertia high. In a heavy sea or in diving with the dome extended, the dome as a whole will be deflected. Thus the dome should be short with a wide base, favoring the cylindrical shape. High lift and buoyancy forces must be overcome in retracting the dome. In rough seas the exposure time can be expected to be between 38 and 96 seconds, depending on the heading with respect to sea.

## ACKNOWLEDGMENT

The author is indebted to Mrs. Margaret Bledsoe, of the Seaworthiness Branch, who reviewed the portions of this report relating to wave action.

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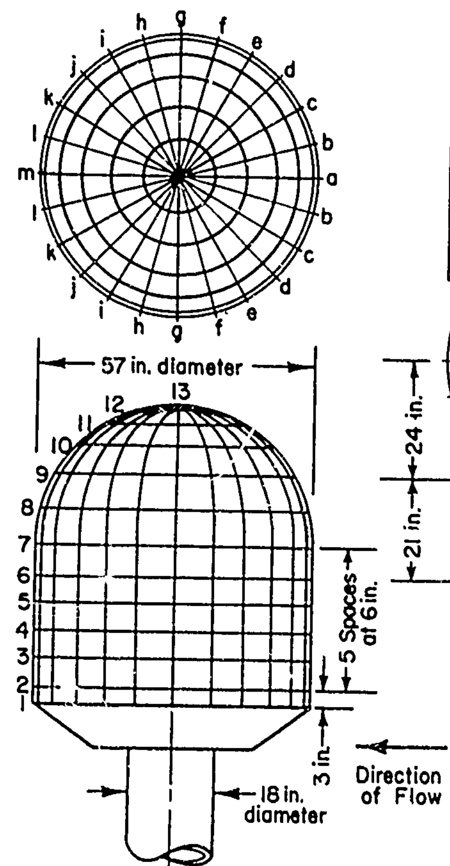


Figure 1a - Cylindrical Dome

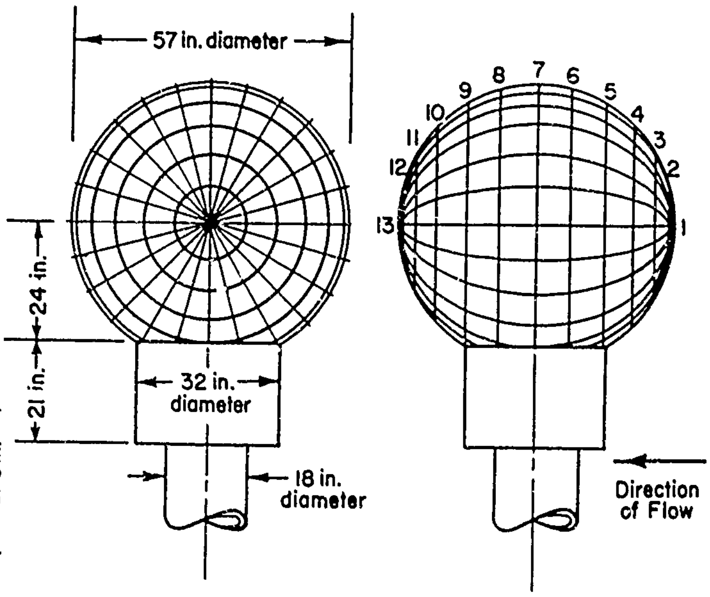


Figure 1b - Spherical Dome

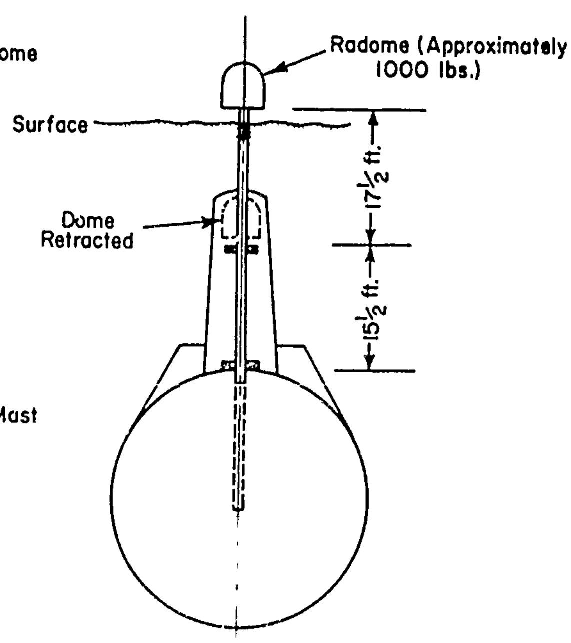


Figure 1c - Radome-Mast Configuration

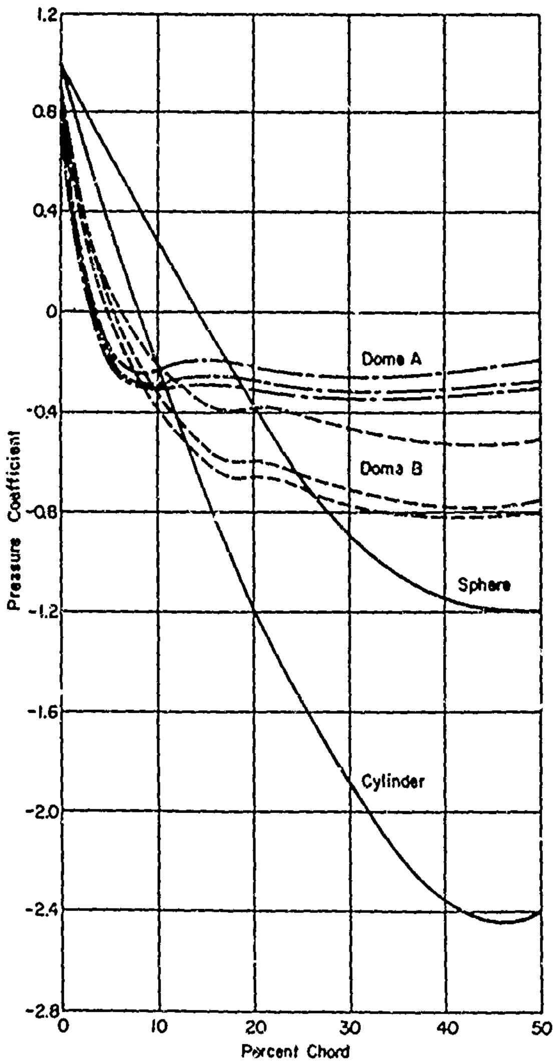


Figure 2a – Pressure Distributions

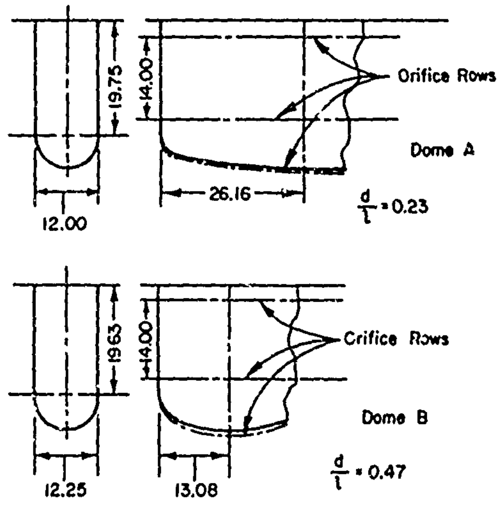


Figure 2b – Dome Shapes

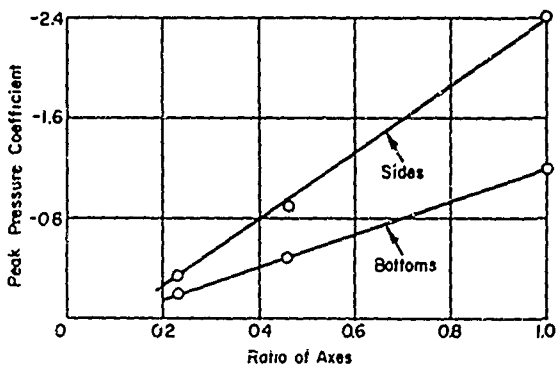


Figure 2c – Maximum Negative Pressure Coefficients

TABLE 1

Force Distribution Over Cylindrical Radome

6 Knots

| Station | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a | 28.3 | 30.3 | 32.2 | 32.2 | 32.2 | 32.2 | 34.1 | 17.4 | - 2.9 | -14.0 | -11.4 | |
| b | 15.1 | 19.0 | 23.2 | 23.2 | 24.5 | 28.9 | 18.2 | 4.9 | - 5.7 | -16.0 | -11.4 | |
| c | 4.2 | 3.5 | 3.2 | 3.2 | 8.1 | 10.8 | 5.0 | - 4.5 | -11.4 | -18.0 | -12.0 | |
| d | 0 | -11.3 | -22.5 | -22.5 | -17.7 | -14.4 | 0 | -13.9 | -22.8 | -20.0 | -12.5 | |
| e | -36.7 | -43.5 | -50.2 | -50.2 | -43.5 | -40.8 | -44.1 | -39.9 | -33.1 | -23.8 | -12.8 | |
| f | -52.4 | -63.1 | -74.1 | -74.1 | -64.4 | -61.4 | -62.7 | -53.4 | -41.3 | -27.0 | -13.1 | |
| g | -54.7 | -66.0 | -77.3 | -77.3 | -67.6 | -65.0 | -65.8 | -55.5 | -42.8 | -28.0 | -13.5 | 37.8 |
| h | -48.6 | -58.0 | -67.0 | -67.0 | -58.6 | -56.7 | -58.4 | -50.3 | -39.3 | -26.4 | -13.1 | |
| i | -20.6 | -18.3 | -16.1 | -16.1 | -16.4 | -19.1 | -24.8 | -26.0 | -24.5 | -20.6 | -12.5 | |
| j | - 6.4 | - 3.2 | 0 | 0 | 0 | -10.1 | -12.0 | -13.9 | -17.1 | -17.6 | -12.1 | |
| k | - 4.8 | - 2.3 | 0 | 0 | 1.0 | 2.2 | - 5.8 | -12.5 | -16.2 | -16.8 | -11.6 | |
| l | - 4.8 | - 2.3 | 0 | 0 | 1.0 | 2.2 | - 5.8 | -12.5 | -16.2 | -11.2 | -11.2 | |
| m | - 3.9 | - 2.0 | 0 | 0 | 1.0 | 2.2 | - 4.6 | - 4.2 | -16.2 | -11.2 | -10.9 | |

TABLE 2

Force Distribution Over Cylindrical Radome

20 Knots

| Station | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| a | 315 | 337 | 358 | 358 | 358 | 402 | 378 | 193 | - 32 | -155 | -127 | |
| b | 168 | 211 | 258 | 258 | 272 | 232 | 202 | 54 | - 63 | -178 | -127 | |
| c | 47 | 39 | 36 | 36 | 90 | - 90 | 56 | - 50 | -127 | -200 | -132 | |
| d | 0 | -125 | -251 | -251 | -197 | -161 | 0 | -154 | -254 | -222 | -138 | |
| | -408 | -483 | -558 | -558 | -483 | -454 | -490 | -444 | -368 | -264 | -141 | |
| f | -580 | -702 | -823 | -823 | -716 | -683 | -697 | -594 | -460 | -300 | -145 | |
| g | -609 | -734 | -859 | -859 | -752 | -724 | -731 | -618 | -476 | -311 | -150 | -420 |
| h | -541 | -644 | -744 | -744 | -652 | -631 | -650 | -560 | -437 | -293 | -145 | |
| i | -229 | -204 | -179 | -179 | -183 | -213 | -275 | -290 | -273 | -229 | -138 | |
| j | - 72 | - 36 | 0 | 0 | 0 | -113 | -133 | -154 | -190 | -195 | -133 | |
| k | - 54 | - 25 | 0 | 0 | 11 | 24 | - 65 | -140 | -181 | -186 | -129 | |
| l | - 54 | - 25 | 0 | 0 | 11 | 24 | - 65 | -140 | -181 | -124 | -124 | |
| m | - 43 | - 21 | 0 | 0 | 11 | 24 | - 52 | - 46 | -181 | -124 | -121 | |

TABLE 3

Force Distribution Along Element of Spherical Radome

| Section | Force at 6 Knots lb | Force at 20 Knots lb |
|---|---|---|
| 1 | 31.5 | 350 |
| 2 | 9.4 | 104 |
| 3 | 9.4 | 105 |
| 4 | - 2.9 | - 32 |
| 5 | -22.2 | -246 |
| 6 | -40.6 | -451 |
| 7 | -48.1 | -534 |
| 8 | -39.5 | -438 |
| 9 | -19.4 | -216 |
| 10 | 0 | 0 |
| 11 | 2.6 | 29 |
| 12 | 1.4 | 15 |
| 13 | 4.1 | 46 |

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